POLITECNICO
MILANO 1863

Prova finale: Introduzione all'analisi di missioni spaziali

Docente: **Topputo Francesco**

Elaborato: n. 13

Pucciarelli Antonio
10619018

Tabbó Giuseppe Emanuele
10614332

Tirotta Raffaele
10616178

Anno accademico **2019/2020**

Contents

1 Introduction

The purpose of this work is to transfer a satellite from the initial point of the assigned orbit to the final position on a second orbit. The initial point is described by position and velocity given in the celestial reference system, whereas the final orbit is characterised by its orbital parameters.

First of all, it is necessary to switch from one reference frame to another in order to characterise both the orbit and the position of the satellite, then the aim is accomplished with a standard and an alternative transfer.

The standard strategy is composed by a plane change, a change of pericentre argument and one of the possible bitangent transfers, that are PA, AP, AA, and PP.

The analysis of the results about the previous strategy led to evaluating other types of transfer in order to minimize the total time and cost of manoeuvre.

2 Initial orbit characterisation

2.1

Here is the given position and velocity:

vector	$\boldsymbol{i}_x$	$\boldsymbol{i}_y$	$\boldsymbol{i}_z$	UDM
$\boldsymbol{r}_i$	-8485.9649	1373.8778	2665.1050	$[km]$
$\boldsymbol{v}_i$	-2.1880	-6.0200	-1.5240	$\left[\frac{km}{s}\right]$

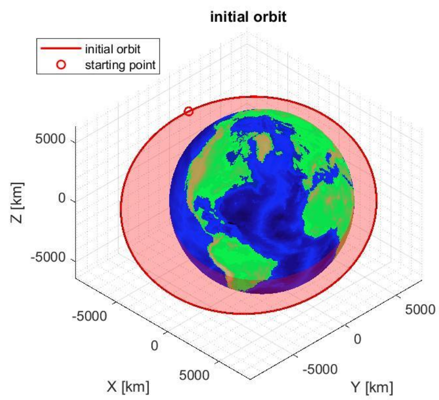
Then is represented in orbital parameters:

$\boldsymbol{a}_i$	$\boldsymbol{e}_i$	$\boldsymbol{i}_i$	$\boldsymbol{\Omega}_i$	$\boldsymbol{\omega}_i$	$\boldsymbol{\Theta}_i$
$8813.4\ km$	0.1073	$0.4080\ rad$	$0.6393\ rad$	$0.4253\ rad$	$1.8738\ rad$

2.2

The initial orbit has a low eccentricity and it is MEO. Because of the small eccentricity the velocity range along the orbit is tight.

2.3



3 Final orbit characterisation

3.1

Here are the given orbital parameters:

a_f	e_f	i_f	Ω_f	ω_f	Θ_f
16160 km	0.335	0.6959 rad	0.503 rad	0.8538 rad	1.825 rad

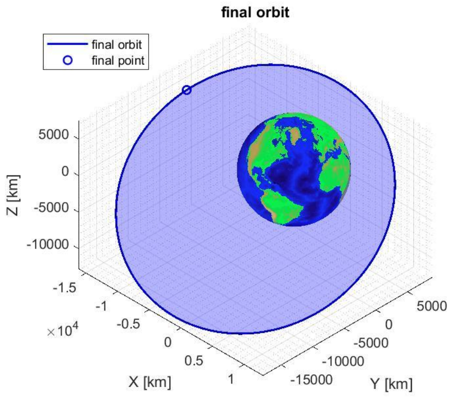
Then is represented in position and velocity:

vector	i_x	i_y	i_z	UDM
r_i	-14869.5988	-2054.5991	4483.7968	$[km]$
v_i	-1.9123	-4.1673	-2.2798	$\left[\frac{km}{s}\right]$

3.2

The final orbit lays on a different plane. It has a higher semi axis than the previous orbit. This leads to a longer period and bigger dimensions than the initial orbit.

3.3



4 Transfer trajectory definition and analysis

4.1

First of all, it has to find the intersection between the initial orbit and the final plane. At this point the satellite performs a plane change manoeuvre that is in common with all the 4 standard strategies. The next step consists in a pericentre change manoeuvre and there are two ways for doing it. This depends on the standard strategy transfer:

- for PP and AA, the new pericentre anomaly is $\omega_f + \pi$
- for PA and AP, the new pericentre anomaly is ω_f

Then the bitangent manoeuvre takes place at the apsidal points. At the end, the satellite reaches the final point on the final orbit.

4.2

For each transfer there are two points where it is possible to change plane and pericentre anomaly.
The choices are done as follows:

- minimizing the cost of manoeuvre when changing plane
- minimizing the manoeuvre time when changing anomaly pericentre because the choice has no influence on manoeuvre cost (please note: the change pericentre argument point is different for each transfer).

$\boldsymbol{\Delta v}$ $\left[\frac{km}{s}\right]$	**AP**	**PA**	**PP**	**AA**
Change plane	0.8357	1.8357	1.8357	1.8357
Change PerArg	0.2249	0.2249	1.4341	1.4341
$\boldsymbol{\Delta v}_1$	0.5046	1.1268	0.1585	1.4623
$\boldsymbol{\Delta v}_2$	1.0949	0.3628	1.4373	0.1127
$\boldsymbol{\Delta v}_{TOT}$	3.6603	3.5503	4.8656	4.8438

Δt $[h]$	**AP**	**PA**	**PP**	**AA**
Δt_1	0.2173	0.2173	0.2173	0.2173
Δt_2	0.3902	1.5095	2.0146	1.0247
Δt_3	0.0697	0.0454	0.5505	0.7046
Δt_4	1.4349	2.4687	1.2411	2.7104
Δt_5	1.0356	3.8752	1.0357	3.8752
Δt_{TOT}	3.1479	8.1162	5.0591	8.5325

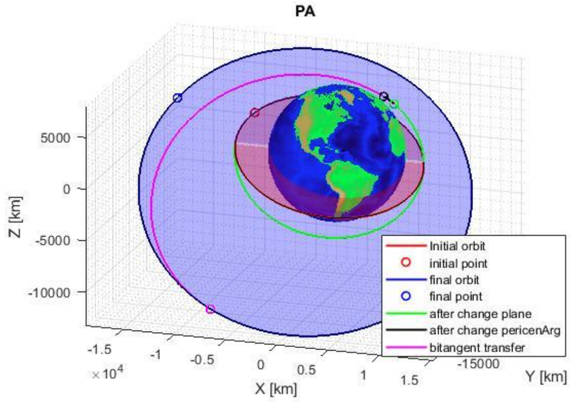
Each other case is reported in A.

Doing the maths, it is important to analyse the results. First of all, it is possible to get the minimum $\boldsymbol{\Delta v}$ for PA and minimum Δt for AP.

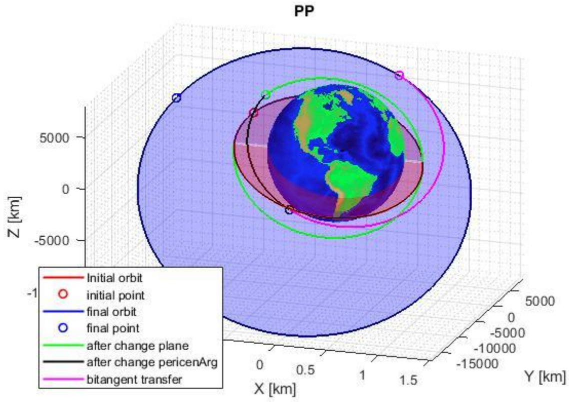
AA and PP are characterised by the highest cost for realising the manoeuvre that change pericentre argument, because of the fact it is necessary to guarantee a higher $\Delta\omega$ compared to AP and PA strategy.

Another consideration is about Δt. As the values show, it is bigger for an orbital far from the attractor. For this reason, it increases because the satellite stays a much longer time on the final orbit.

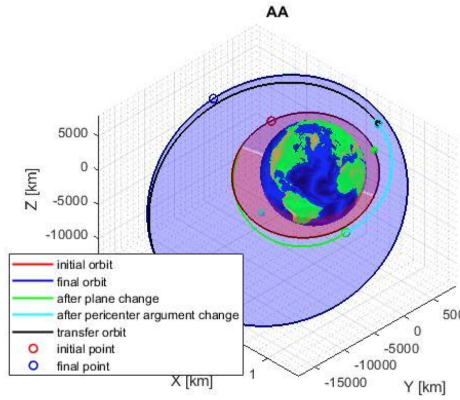
4.3



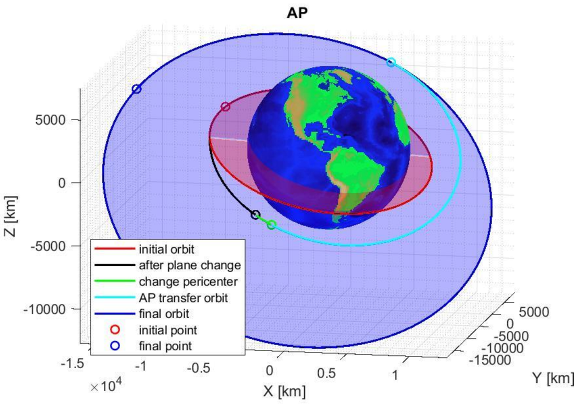
(a) PA



(b) PP



(c) AA



(d) AP

4.4

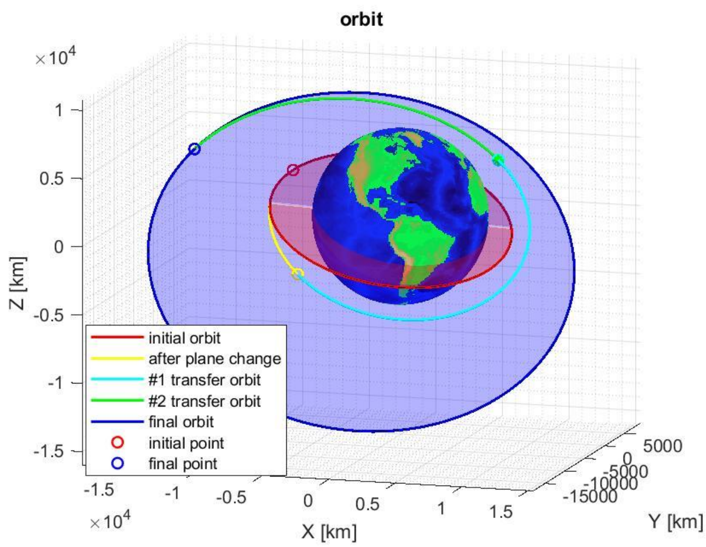
4.4.1 Qua

The following strategy is made by 4 manoeuvres.

As in the standard strategies, when it has reached the intersection point between the initial orbit and the final plane there is a plane change.

This is the most expensive manoeuvre among the 4 but it is the one that minimizes the total cost for this set of manoeuvres.

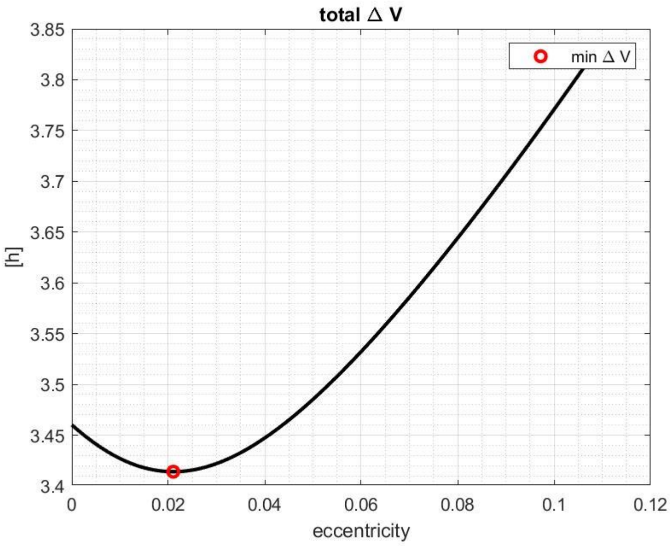
The first manoeuvre takes place at the new orbit apogee and it changes only the satellite orbit geometry. If the change plane were made after this transfer the total manoeuvre cost would have been higher for any set of manoeuvres.



Once reached the 1st transfer orbit perigee there will be another transfer manoeuvre. This manoeuvre is figured out imposing:

- the perigee must be coincident for the two transfer manoeuvres
- the passage of the 2nd transfer orbit at the final orbit target point

The 2nd transfer orbit passes at the target point, but it is not tangent to the final orbit.



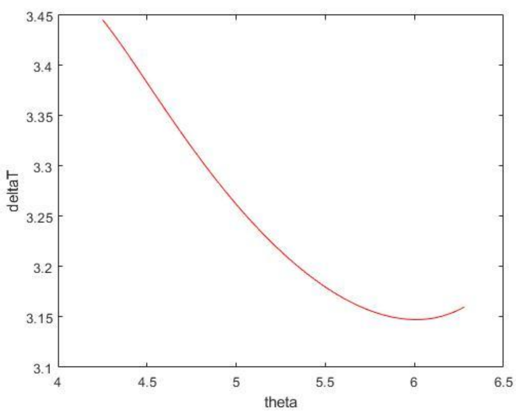
When the satellite is on the target, there will be the last manoeuvre that changes the satellite orbit from the 2nd transfer orbit to the final orbit. This change of velocity is made by varying the radial and transversal velocity of the satellite.

This strategy is made by imposing different orbital shapes for the 1st transfer orbit. The best geometry for the 1st transfer orbit is the one that minimizes the manoeuvre cost. The 2nd orbit geometry is constrained by the 1st transfer orbit geometry, by its perigee, and by the target point.

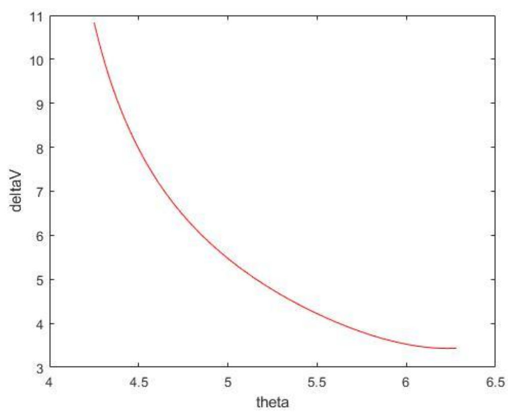
There is an important aspect to mark up: the second transfer orbit and final orbit allow feasible manoeuvring at the target point because of the small difference in velocity direction by the two orbits at that point. If the velocity for the final orbit and for the transfer orbit at the target point were too diverse, it would not be so convenient manoeuvring at the target point; this is because the manoeuvre cost would be too high.

To sum up, the time to complete the manoeuvres Δt is 3.94 hours and the total cost Δv is $3.41\frac{km}{s}$.

4.4.2 Quo



(a)



(b)

The following transfer strategy has as main purpose the minimization of the amount of manoeuvres done to reach the target.

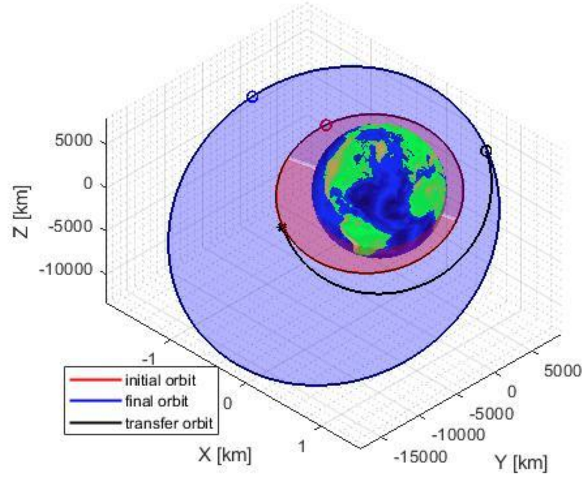
After the plane change, the satellite reaches the initial orbit's apocenter and from here it passes on a transfer orbit that leads it to the final orbit.

At the end, the satellite passes on the final orbit at an arbitrary true anomaly value.

So, the transfer orbit must satisfy two conditions (one of these is set, whereas the other can change):

- its pericentre coincides with the initial orbit's apocenter.
- there is at least a point on final orbit at a certain value of Θ where final and transfer orbit intersect.

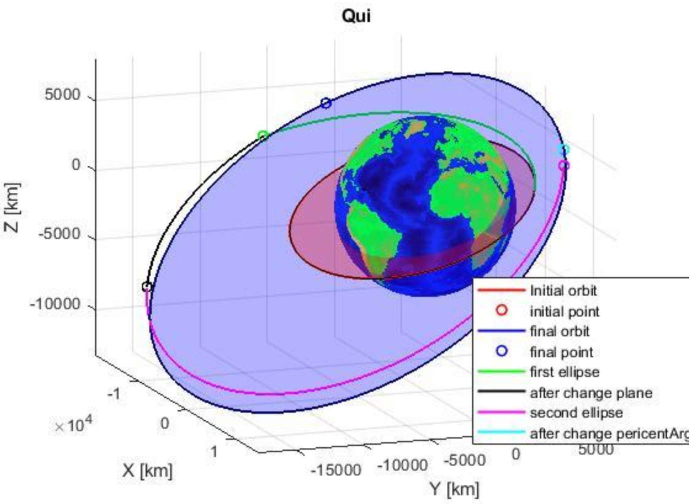
Changing the that satisfy the second condition leads to different transfer orbit and, consequently, to different values of Δt and Δv, as follows: (the values of Θ refer to the final orbit true anomaly and their range allows to obtain ellipticals transfer orbits only)



Due to the facts that the minimization of cost is always preferred and the variation of Δt is very little around $\theta = 6rad$, it is better to choose the transfer that minimizes Δv ($\Delta v = 3.4391\frac{km}{s}$, $\Delta t = 3.16h$). The transfer orbit is the following in black:

4.4.3 Qui

From data in 4.2 the most expensive manoeuvre is the plane change. To reduce it, a solution could be to make it far away from the attractor. The problem is that the Δi is too low to use a bielliptic transfer from the theory. Because of a low ratio of $\frac{r_{pf}}{r_{pi}}$ the coplanar bielliptic transfer is expensive too. The idea is to combine the two effects and to do the change plane on the bielliptic orbit, in order to go far away from the attractor to reduce the cost of this manoeuvre. After that there is a change in pericenter argument, to get the final one.



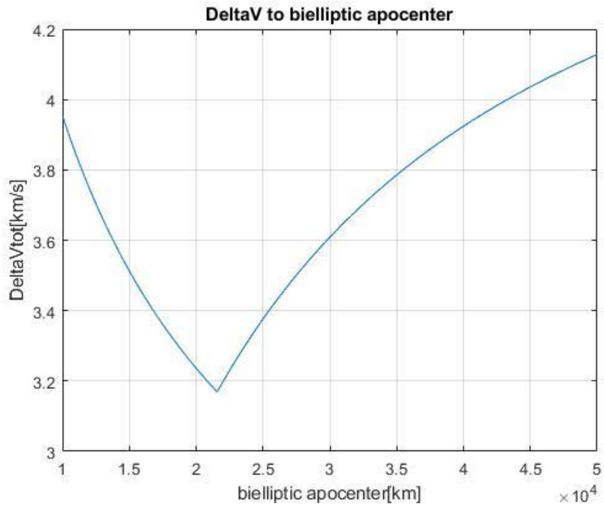
At first it is necessary to take a vector of initial guess for the apocenter of the orbit transfer and find the value for which the Δv is minimized. The result is that this value is coincident with the apogee of the final orbit, as it is shown in the graph. The reason is that the cost of bielliptic transfer increases a lot compared to the gain in the change. So, it becomes a change plane on a bitangent transfer PA. The other Δv are the same as PA, except for the change plane, which decreases to $1.1320\frac{km}{s}$.

Anyway, the idea leads to a $\Delta v = 3.1689\frac{km}{s}$ lower than the standard strategy. Obviously, the time increases, for the reasons discussed before, to $\Delta t = 8.1576h$.

5 Conclusions

First of all, the fact that the transfer orbit semi-axis for AA manoeuvre is bigger than the others leads to a waste of time that is not justified by a better cost, so AA is the worst possible manoeuvre. The change pericentre argument manoeuvre is more demanding for the AA and PP manoeuvres, this is because of the high $\Delta\omega$. The best transfer manoeuvre in terms of Δt is the AP strategy, for Δv is PA. Between AP and PA there is a huge difference in time, this brings to the choice of AP manoeuvre because it has the lowest Δt on Δv.

The nonstandard manoeuvres are made up to improve some aspects of the previous strategies.



Looking for minimize the cost, the best solution among all the strategies is ‘Qui’, but the time increases a lot.

The ‘Quo’ minimizes the amount of manoeuvres, but the fact that transfer orbit geometry is strictly like the AP one makes ‘Quo’ $\frac{\Delta t}{\Delta v}$ really advantageous.

The ‘Qua’ manoeuvre figures out to minimize the velocity using a manoeuvre at the target point. Good results are achievable because of the geometry similarity between the last transfer manoeuvre orbit and the final orbit. In addition to the good result in velocity minimization, the ‘Qua’ strategy gets good results with total transfer time too. The total time for the ‘Qua’ manoeuvre is guaranteed by the low distance from the attractor.

It is impossible to get the best strategy, what really matters is choice.

A Appendix

Table 1: Transfer AP (standard strategy) [plane change 2; change pericenter argument 2]

t $[s]$	a $[km]$	e $[-]$	i $[rad]$	Ω $[rad]$	ω $[rad]$	θ $[rad]$	Δv $\left[\frac{km}{s}\right]$
0	8.81 e+3	0.1073	0.408	0.6393	0.4253	1.8737	-
7.824 e+2	8.81 e+3	0.1073	0.408	0.6393	0.4253	2.4131	1.8357
	8.81 e+3	0.1073	0.695	0.5030	0.5426	2.4131	
2.187 e+3	8.81 e+3	0.1073	0.695	0.5030	0.5426	3.2971	0.2249
	8.81 e+3	0.1073	0.695	0.5030	0.8538	2.9860	
2.438 e+3	8.81 e+3	0.1073	0.695	0.5030	0.8538	3.1415	0.5046
	1.02 e+4	0.3350	0.695	0.5030	0.8538	3.1415	
7.604 e+3	1.02 e+4	0.3350	0.695	0.5030	0.8538	6.2831	1.0949
	1.61 e+4	0.3350	0.695	0.5030	0.8538	6.2831	
1.133 e+4	1.61 e+4	0.3350	0.695	0.5030	0.8538	1.8250	-

Table 2: Transfer AA 11(standard strategy)

t $[s]$	a $[km]$	e $[-]$	i $[rad]$	Ω $[rad]$	ω $[rad]$	θ $[rad]$	Δv $\left[\frac{km}{s}\right]$
0	8813.4	0.1073	0.4080	0.6393	0.4253	1.8737	-
7.824 e+2	8813.4	0.1073	0.4080	0.6393	0.4253	2.4130	1.8357
	8813.4	0.1073	0.6959	0.5030	0.5426	2.4130	
4472.5	8813.4	0.1073	0.6959	0.5030	0.5426	4.8680	1.4341
	8813.4	0.1073	0.6959	0.5030	3.9954	1.4151	
7009	8813.4	0.1073	0.6959	0.5030	3.9954	3.1415	1.4623
	15666	0.3771	0.6959	0.5030	0.8538	0.00	
16766	15666	0.3771	0.6959	0.5030	0.8538	3.1415	0.1127
	16160	0.3350	0.6959	0.5030	0.8538	3.1415	
30717	16160	0.3350	0.6959	0.5030	0.8538	1.8250	-

Table 3: Transfer PA(standard strategy) 11

t $[s]$	a $[km]$	e $[-]$	i $[rad]$	Ω $[rad]$	ω $[rad]$	θ $[rad]$	Δv $\left[\frac{km}{s}\right]$
0	8.81 e+3	0.1073	0.4080	0.6393	0.4253	1.8737	-
782.4	8.81 e+3	0.1073	0.4080	0.6393	0.4253	2.4131	1.8357
	8.81 e+3	0.1073	0.6959	0.5030	0.5426	2.4131	
6216.8	8.81 e+3	0.1073	0.6959	0.5030	0.5426	0.1556	0.2249
	8.81 e+3	0.1073	0.6959	0.5030	0.8538	6.1276	
6380.4	8.81 e+3	0.1073	0.6959	0.5030	0.8538	0.00	1.1268
	14721	0.4655	0.6959	0.5030	0.8538	0.00	
15268	14721	0.4655	0.6959	0.5030	0.8538	3.1415	0.3628
	16160	0.3350	0.6959	0.5030	0.8538	3.1415	
29218	16160	0.3350	0.6959	0.5030	0.8538	1.8250	-

Table 4: Transfer PP(standard strategy) 11

$t\ [s]$	$a\ [km]$	$e\ [-]$	$i\ [rad]$	$\Omega\ [rad]$	$\omega\ [rad]$	$\theta\ [rad]$	$\Delta v\ \left[\frac{km}{s}\right]$
0	8.81 e+3	0.1073	0.4080	0.6393	0.4253	1.8737	-
7.824 e+2	8.81 e+3	0.1073	0.4080	0.6393	0.4253	2.4131	1.8357
	8.81 e+3	0.1073	0.6959	0.5030	0.5426	2.4131	
8.035 e+3	8.81 e+3	0.1073	0.6959	0.5030	0.5426	1.7264	1.4341
	8.81 e+3	0.1073	0.6959	0.5030	3.9954	4.5568	
1.001 e+4	8.81 e+3	0.1073	0.6959	0.5030	3.9954	0.00	0.1585
	9.30 e+3	0.1547	0.6959	0.5030	3.9954	0.00	
1.448 e+4	9.30 e+3	0.1547	0.6959	0.5030	3.9954	3.1415	1.4373
	1.61 e+4	0.3350	0.6959	0.5030	0.8538	0.00	
1.821 e+4	1.61 e+4	0.3350	0.6959	0.5030	0.8538	1.8250	-

Table 5: Transfer Qui

$t\ [s]$	$a\ [km]$	$e\ [-]$	$i\ [rad]$	$\Omega\ [rad]$	$\omega\ [rad]$	$\theta\ [rad]$	$\Delta v\ \left[\frac{km}{s}\right]$
0	8.81 e+3	0.1073	0.4080	0.6393	0.4253	1.8737	-
6053.2	8.81 e+3	0.1073	0.4080	0.6393	0.4253	0.00	1.1268
	14721	0.4655	0.4080	0.6393	0.4253	0.00	
10566	14721	0.4655	0.4080	0.6393	0.4253	2.4131	1.1320
	14721	0.4655	0.6959	0.5030	0.5426	2.4131	
14941	14721	0.4655	0.6959	0.5030	0.5426	3.1415	0.3628
	16160	0.3350	0.6959	0.5030	0.5426	3.1415	
25401	16160	0.3350	0.6959	0.5030	0.5426	0.1556	0.5473
	16160	0.3350	0.6959	0.5030	0.8538	6.1276	
29368	16160	0.3350	0.6959	0.5030	0.8538	1.8250	-

Table 6: Transfer Quo

$t\ [s]$	$a\ [km]$	$e\ [-]$	$i\ [rad]$	$\Omega\ [rad]$	$\omega\ [rad]$	$\theta\ [rad]$	$\Delta v\ \left[\frac{km}{s}\right]$
0	8813.4	0.1073	0.408	0.6393	0.4253	1.8737	-
782.4	8813.4	0.1073	0.408	0.6393	0.4253	2.4100	1.8357
	8813.4	0.1073	0.695	0.5030	0.5426	2.4100	
1947.1	8813.4	0.1073	0.695	0.5030	0.5426	3.1415	0.5084
	10266	0.0494	0.695	0.5030	3.684	0.00	
7681.6	10266	0.0494	0.695	0.5030	3.684	3.4497	1.0950
	16160	0.3350	0.695	0.5030	0.8538	6.2830	
11376	16160	0.3350	0.695	0.5030	0.8538	1.8250	-

Table 7: Transfer Qua

t $[s]$	a $[km]$	e $[-]$	i $[rad]$	Ω $[rad]$	ω $[rad]$	θ $[rad]$	Δv $\left[\frac{km}{s}\right]$
0	8.81 e+3	0.1073	0.408	0.6393	0.4253	1.8737	-
7.824 e+2	8.81 e+3	0.1073	0.408	0.6393	0.4253	2.4131	1.8357
	8.81 e+3	0.1073	0.695	0.5030	0.5426	2.4131	
1.936 e+3	8.81 e+3	0.1073	0.695	0.5030	0.5426	3.1415	0.2850
	9.55 e+3	0.0210	0.695	0.5030	0.5426	3.1415	
6.585 e+3	9.55 e+3	0.0210	0.695	0.5030	0.5426	0.00	1.0036
	1.45 e+4	0.3554	0.695	0.5030	0.5426	0.00	
1.059 e+4	1.45 e+4	0.3554	0.695	0.5030	0.5426	2.1361	0.2896
	1.61 e+4	0.0481	0.695	0.5030	0.8538	1.8250	
1.059 e+4	1.61 e+4	0.0481	0.695	0.5030	0.8538	1.8250	-